IYGB - MP1 PAPER Q - QUESTION 1

METHOD A

- $a^3 - a + 1 = a(a^2-1) + 1 = a(a+1)(a-1) + 1$

- AS $a(a+1)(a-1)$ CONTAINS CONSECUTIVE INTEGERS, AT LEAST ONE OF THEM WILL BE EVEN, SO $a(a+1)(a-1)$ WILL BE EVEN FOR ALL $a \in \mathbb{N}$

- HENCE $a(a+1)(a-1)+1$ WILL BE ODD FOR ALL $a \in \mathbb{N}$

METHOD B (BY EXHAUSTION)

- LET a BE EVEN, $a = 2n$

$$(2n)^3 - 2n + 1 = 8n^3 - 2n + 1 = 2(4n^3 - n) + 1$$
$$= 2m + 1$$
$$\therefore \text{ODD}$$

- LET a BE ODD, $a = 2n+1$

$$(2n+1)^3 - (2n+1) + 1 = 8n^3 + 12n^2 + 6n + 1 - 2n - 1 + 1$$
$$= 8n^3 + 12n^2 + 4n + 1$$
$$= 2[4n^3 + 6n^2 + n] + 1$$
$$= 2m + 1$$
$$\therefore \text{ODD}$$

HENCE $a^3 - a + 1$ IS ODD FOR $a \in \mathbb{N}$

IYGB - MP1 PAPER Q - QUESTION 2

PROCEED BY INTEGRATING THE L.H.S OF THE EQUATION

$$\Rightarrow \int_1^3 6x^2 + kx \, dx = 8$$

$$\Rightarrow \left[2x^3 + \frac{1}{2}kx^2 \right]_1^3 = 8$$

$$\Rightarrow \left(2 \times 3^3 + \frac{1}{2}k \times 3^2\right) - \left(2 \times 1^3 + \frac{1}{2}k \times 1^2\right) = 8$$

$$\Rightarrow 54 + \frac{9}{2}k - 2 - \frac{1}{2}k = 8$$

$$\Rightarrow 52 + 4k = 8$$

$$\Rightarrow 4k = -44$$

$$\Rightarrow k = -11$$

IYGB – MP1 PAPER Q – QUESTION 3

THE DEFINITION OF THE DERIVATIVE FOR $y = f(x)$ IS GIVEN BY

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0}\left[\frac{f(x+h) - f(x)}{h}\right]$$

$$f'(x) = \lim_{h \to 0}\left[\frac{(x+h)^2 - x^2}{h}\right]$$

$$f'(x) = \lim_{h \to 0}\left[\frac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}\right]$$

$$f'(x) = \lim_{h \to 0}\left[\frac{2xh + h^2}{h}\right]$$

DIVIDING h FROM ALL THE 3 TERMS

$$f'(x) = \lim_{h \to 0}\left[2x + h\right]$$

AS h TENDS TO ZERO THE ONLY TERM THAT SURVIVES IS $2x$

$$f'(x) = 2x$$

// AS REQUIRED

IYGB - MP1 PAPER Q - QUESTION 4

(a) FORM AN EQUATION USING THE POINT $P(55, 1)$

$\Rightarrow y = 2\sin(2x+k)^\circ$

$\Rightarrow 1 = 2\sin(110+k)^\circ$

$\Rightarrow \sin(k+110)^\circ = \frac{1}{2}$

$\arcsin(\frac{1}{2}) = 30^\circ$

$\Rightarrow \begin{cases} k+110 = 30 \pm 360n \\ k+110 = 150 \pm 360n \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} k = -80 \pm 360n \\ k = 40 \pm 360n \end{cases}$

$\Rightarrow k = \begin{cases} \ldots, \cancel{-440}, \cancel{-80}, \cancel{280}, \cancel{640}, \ldots \\ \ldots, \cancel{-320}, 40, \cancel{400}, \cancel{760}, \ldots \end{cases}$

$\therefore$ ONLY VALUE IN THE REQUIRED RANGE $0 < k < 90$ IS $k = 40$

b) USING $k = 40$ & THE POINT $Q(\alpha, \sqrt{3})$

$\Rightarrow y = 2\sin(2x+40)^\circ$

$\Rightarrow \sqrt{3} = 2\sin(2\alpha+40)^\circ$

$\Rightarrow \sin(2\alpha+40)^\circ = \frac{\sqrt{3}}{2}$

$\arcsin(\sqrt{3}/2) = 60$

$\Rightarrow \begin{cases} 2\alpha+40 = 60 \pm 360n \\ 2\alpha+40 = 120 \pm 360n \end{cases}$ $n = 0, 1, 2, 3, \ldots$

$\Rightarrow \begin{cases} 2\alpha = 20 \pm 360n \\ 2\alpha = 80 \pm 360n \end{cases}$

$\Rightarrow \begin{cases} \alpha = 10 \pm 180n \\ \alpha = 40 \pm 180n \end{cases}$

IN THE RANGE $0 < \alpha < 360$

$\alpha = 10, 190, 40, 220$

IYGB — MP1 PAPER Q — QUESTION 5

a) "TAKING LOGS", BASE 10, FOR THE EQUATION

$$\Rightarrow y = a \times k^x$$

$$\Rightarrow \log_{10} y = \log_{10}(a \times k^x)$$

$$\Rightarrow \log_{10} y = \log_{10} a + \log_{10} k^x$$

$$\Rightarrow \log_{10} y = \log_{10} a + x\log_{10} k$$

$$\Rightarrow \log_{10} y = (\log_{10} k)x + (\log_{10} a)$$

$$Y = m \quad x \quad c$$

∴ A LINEAR RELATIONSHIP INDEED

b) LOOKING AT THE y INTERCEPT, B (0, 3)

$$\Rightarrow \log_{10} a = 3$$

$$\Rightarrow a = 10^3$$

$$\Rightarrow a = 1000$$

LOOKING AT THE GRADIENT OF THE LINE THROUGH $A(-\frac{3}{2}, 0)$ & $(0, 3)$

$$\Rightarrow \text{"}\frac{y_2 - y_1}{x_2 - x_1}\text{"} = \log_{10} k$$

$$\Rightarrow \frac{3 - 0}{0 - (-\frac{3}{2})} = \log_{10} k$$

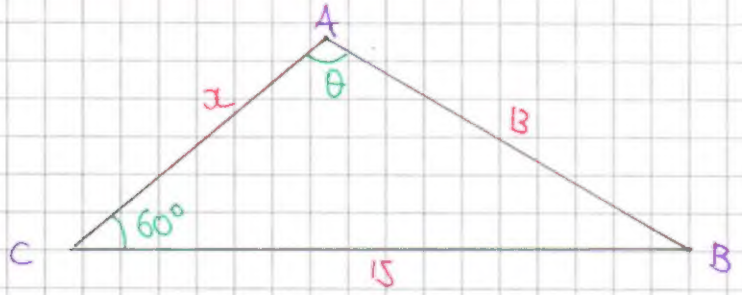
$$\Rightarrow 2 = \log_{10} k$$

$$\Rightarrow k = 10^2$$

$$\Rightarrow k = 100$$

IYGB - MP1 PAPER Q - QUESTION 6

STARTING WITH A DIAGRAM



BY THE SINE RULE

$$\frac{13}{\sin 60^\circ} = \frac{15}{\sin\theta}$$

$$\sin\theta = \frac{15\sin 60}{13}$$

$$\sin\theta = \frac{15}{26}\sqrt{3}$$

$$\theta = \begin{cases} 87.7957...^\circ \\ 92.2042...^\circ \end{cases}$$

BY THE COSINE RULE

$$|AC|^2 = |AB|^2 + |BC|^2 - 2|AB||BC|\cos\theta$$

$$|AC|^2 = 13^2 + 15^2 - 2\times13\times15\times\cos\theta$$

$$|AC| = \sqrt{394 - 390\cos\theta}$$

$$|AC| = \begin{cases} \text{IF } \theta = 87.7957...^\circ, & x = 8 \\ \text{IF } \theta = 92.2042..., & x = 7 \end{cases}$$

IYGB - MP1 PAPER Q - QUESTION 7

a) START WITH THE GRADIENT OF AB , A(2,1) & B(4,0)

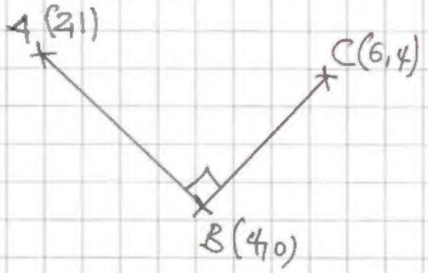
$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-1}{4-2} = \frac{-1}{2}$$

EQUATION OF A LINE PARALLEL TO AB , THROUGH C(6,4)

$$y - y_0 = m(x - x_0)$$
$$y - 4 = -\tfrac{1}{2}(x-6)$$
$$2y - 8 = -x + 6$$
$$2y + x = 14$$

b) FIND THE GRADIENT OF BC

$$m_{BC} = \frac{4-0}{6-4} = \frac{4}{2} = 2$$



AS THE GRADIENTS OF BC & AB ARE NEGATIVE RECIPROCALS OF ONE ANOTHER $\hat{ABC} = 90°$

c) USING THE DISTANCE FORMULA

$$d = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$$

$$|AC| = \sqrt{(4-1)^2 + (6-2)^2}$$

$$|AC| = \sqrt{9 + 16}$$

$$|AC| = 5$$

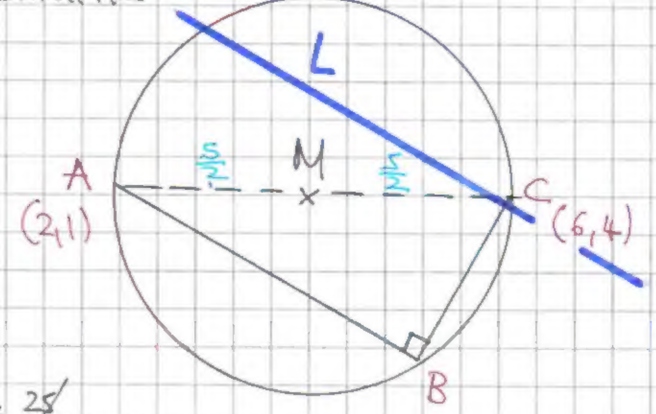
IYGB - MP1 PAPER Q - QUESTION 7

d) LOOKING AT THE DIAGRAM BELOW

BY "CIRCLE THEOREM", AC IS A DIAMETER

$M\left(\frac{6+2}{2}, \frac{1+4}{2}\right)$

$M\left(4, \frac{5}{2}\right)$



$\Rightarrow (x-4)^2 + \left(y-\frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2$

$\Rightarrow x^2 - 8x + 16 + y^2 - 5y + \cancel{\frac{25}{4}} = \cancel{\frac{25}{4}}$

$\Rightarrow x^2 - 8x + 16 + y^2 - 5y = 0$

$\Rightarrow x^2 + y^2 - 8x - 5y + 16 = 0$ // AS REQUIRED

e) LOOKING AT THE ABOVE DIAGRAM FOR L, & SOLVING SIMULTANEOUSLY

$x + 2y = 14$

$x = 14 - 2y$ $\Rightarrow (14-2y)^2 + y^2 - 8(14-2y) - 5y + 16 = 0$

$\Rightarrow 196 - 56y + 4y^2 + y^2 - 112 + 16y - 5y + 16 = 0$

$\Rightarrow 5y^2 - 45y + 100 = 0$

$\Rightarrow y^2 - 9y + 20 = 0$

$\Rightarrow (y-4)(y-5) = 0$

$\Rightarrow y = \begin{cases} 4 \\ 5 \end{cases} \Rightarrow x = \begin{cases} 6 \\ 4 \end{cases}$

$\therefore$ ~~(6,4)~~ & $(4,5)$ //

[POINT C]

IYGB – MP1 PAPER Q – QUESTION 8

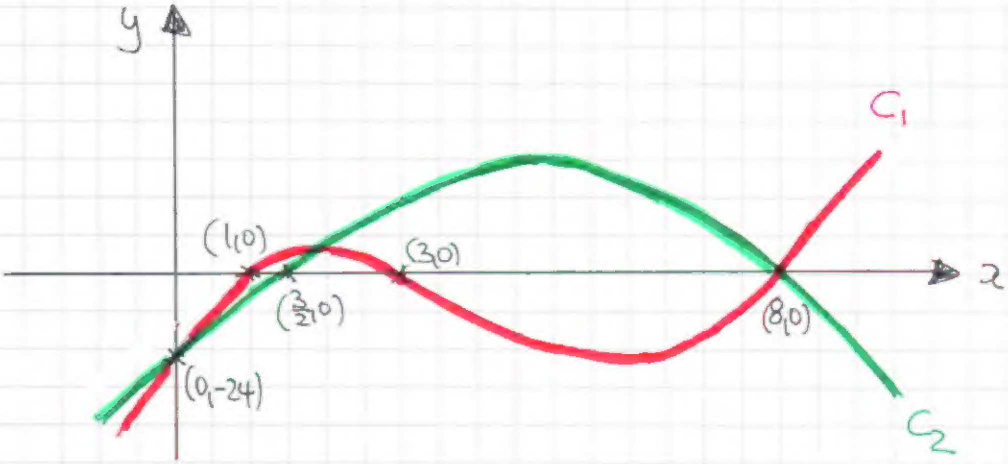
a) C_1: $y = (x-8)(x^2-4x+3)$

$y = (x-8)(x-3)(x-1)$

$(1,0)$, $(3,0)$, $(8,0)$, $(0,-24)$

C_2: $y = (2x-3)(8-x)$

$\left(\frac{3}{2},0\right)$, $(8,0)$, $(0,-24)$



b) SOLVING AS FOLLOWS

$\Rightarrow (x-8)(x^2-4x+3) = (2x-3)(8-x)$

$\Rightarrow (x-8)(x^2-4x+3) - (2x-3)(8-x) = 0$

$\Rightarrow (x-8)(x^2-4x+3) + (x-8)(2x-3) = 0$

$\Rightarrow (x-8)\left[(x^2-4x+\cancel{3}) + (2x-\cancel{3})\right] = 0$

$\Rightarrow (x-8)(x^2-2x) = 0$

$\Rightarrow x(x-2)(x-8)$

$\Rightarrow x = 0, 2, 8$

IYGB - MP1 PAPER Q - QUESTION 8

ALTERNATIVE SOLUTION TO (b)

$\Rightarrow (x-8)(x^2-4x+3) = (2x-3)(8-x)$

$\Rightarrow \begin{array}{l} x^3-4x^2+3x \\ \quad -8x^2+32x-24 \end{array} = 16x-2x^2-24+3x$

$\Rightarrow x^3-12x^2+35x-\cancel{24} = -2x^2+19x-\cancel{24}$

$\Rightarrow x^3-10x^2+16x=0$

$\Rightarrow x(x^2-10x+16)=0$

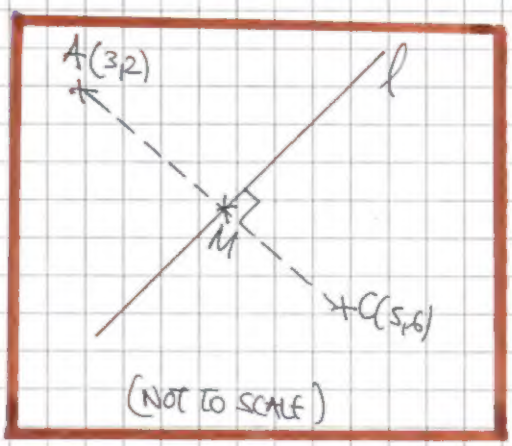
$\Rightarrow x(x-2)(x-8)=0$

$\Rightarrow x = \begin{cases} 0 \\ 2 \\ 8 \end{cases}$

IYGB – MP1 PAPER Q – QUESTION 9

a) GRADIENT OF AC

$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{6-2}{5-3} = \frac{4}{2} = 2$$



MIDPOINT OF AC

$$M\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = M\left(\frac{3+5}{2}, \frac{2+6}{2}\right)$$

$$= M(4,4)$$

EQUATION OF THE PERPENDICULAR BISECTOR l

$$y - y_0 = m(x - x_0)$$

$$y - 4 = -\tfrac{1}{2}(x-4)$$ ← GRADIENT OF l IS NEGATIVE RECIPROCAL OF THE GRADIENT OF AC

$$2y - 8 = -(x-4)$$

$$2y - 8 = -x + 4$$

$$x + 2y = 12$$

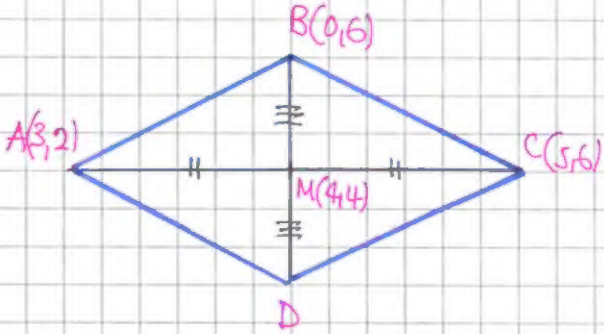
b) FIRST FIND THE CO-ORDINATES OF B

when $x = 0 \implies 2y = 12$

$\implies y = 6$ $\quad\quad \therefore\ B(0,6)$

NEXT DRAW DIAGRAM – M(4,4) MUST ALSO BE THE MIDPOINT OF BD

$$\begin{pmatrix}0\\6\end{pmatrix} \begin{matrix}\xrightarrow{+4}\\ \xrightarrow{-2}\end{matrix} \begin{pmatrix}4\\4\end{pmatrix} \begin{matrix}\xrightarrow{+4}\\ \xrightarrow{-2}\end{matrix} \begin{pmatrix}a\\b\end{pmatrix}$$



$\therefore\ a = 8\ \&\ b = 2$

I.E $D(8,2)$

IYGB - MP1 PAPER Q - QUESTION 9

c) USING DISTANCE FORMULA FOR $P(x_1, y_1)$ & $Q(x_2, y_2)$

$$\boxed{d = |PQ| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}}$$

- $|AM| = \sqrt{(3-4)^2 + (2-4)^2} = \sqrt{1+4} = \sqrt{5}$
- $|BM| = \sqrt{(0-4)^2 + (6-4)^2} = \sqrt{16+4} = \sqrt{20} = 2\sqrt{5}$

AREA OF RHOMBUS CONSISTS OF 4 IDENTICAL RIGHT ANGLED TRIANGLES

$$\text{AREA} = 4 \times \tfrac{1}{2}|AM||BM| = 2 \times \sqrt{5} \times 2\sqrt{5} = 4 \times 5 = \underline{20}$$

IYGB - MP1 PAPER Q - QUESTION 10

a) TIDY THE EQUATION INTO INDICIAL FORM AND DIFFERENTIATE

- $y = \dfrac{4x\sqrt{x} + k}{7x} = \dfrac{4\cancel{x}\sqrt{x}}{7\cancel{x}} + \dfrac{k}{7x} = \frac{4}{7}x^{\frac{1}{2}} + \frac{k}{7}x^{-1}$

- $\dfrac{dy}{dx} = \frac{2}{7}x^{-\frac{1}{2}} - \frac{k}{7}x^{-2}$

- $\left.\dfrac{dy}{dx}\right|_{x=\frac{1}{4}} = \frac{2}{7}\left(\frac{1}{4}\right)^{-\frac{1}{2}} - \frac{k}{7}\left(\frac{1}{4}\right)^{-2} = \frac{2}{7}\times 2 - \frac{k}{7}\times 16 = \frac{4}{7} - \frac{16}{7}k$

b) REARRANGE THE EQUATION OF THE LINE TO "READ" THE GRADIENT

$$\Rightarrow 7y + 44x - 5 = 0$$
$$\Rightarrow 7y = -44x + 5$$
$$\Rightarrow y = -\frac{44}{7}x + \frac{5}{7}$$

HENCE THE GRADIENT AT P WILL BE $-\frac{44}{7}$ (PARALLEL)

$$\Rightarrow \frac{4}{7} - \frac{16}{7}k = -\frac{44}{7}$$
$$\Rightarrow 4 - 16k = -44$$
$$\Rightarrow 48 = 16k$$
$$\Rightarrow k = 3$$

FIND THE y CO-ORDINATE OF P

$$y = \frac{4x\sqrt{x} + 3}{7x} = \frac{4\times\frac{1}{4}\times\sqrt{\frac{1}{4}} + 3}{7\times\frac{1}{4}} = \frac{\left(\frac{1}{2} + 3\right)^{\times 4}}{\left(\frac{7}{4}\right)^{\times 4}} = \frac{2+12}{7} = 2$$

EQUATION OF TANGENT AT $P\left(\frac{1}{4}, 2\right)$

$$y - y_0 = m(x - x_0)$$
$$y - 2 = -\frac{44}{7}\left(x - \frac{1}{4}\right)$$
$$7y - 14 = -44x + 11$$

$$\therefore 44x + 7x = 25$$

IYGB – MP1 PAPER Q – QUESTION 11

FORM A CUBIC IN EXPONENTIALS

$$\Longrightarrow 2e^{2x} - 5e^{x} + 3e^{-x} = 4$$

$$\Longrightarrow 2e^{2x} - 5e^{x} + \frac{3}{e^{x}} = 4$$

MULTIPLY THROUGH BY e^{x}

$$\Longrightarrow 2e^{3x} - 5e^{2x} + 3 = 4e^{x}$$

$$\Longrightarrow 2e^{3x} - 5e^{2x} - 4e^{x} + 3 = 0$$

LET $y = e^{x}$

$$\Longrightarrow 2y^{3} - 5y^{2} - 4y + 3 = 0$$

LET $f(y) = 2y^{3} - 5y^{2} - 4y + 3$ & LOOK FOR FACTORS

- $f(1) = 2 - 5 - 4 + 3 \neq 0$
- $f(-1) = -2 - 5 + 4 + 3 = 0$ $\quad\therefore\ (y+1)$ IS A FACTOR

BY LONG DIVISION OR ALGEBRAIC MANIPULATION WE OBTAIN

$$\Longrightarrow 2y^{2}(y+1) - 7y(y+1) + 3(y+1) = 0$$

$$\Longrightarrow (y+1)(2y^{2} - 7y + 3) = 0$$

$$\Longrightarrow (y+1)(2y-1)(y-3)$$

$$\Longrightarrow y = \begin{cases} -1 \\ \frac{1}{2} \\ 3 \end{cases} \quad \text{OR} \quad e^{x} = \begin{cases} \cancel{-1} \quad (e^{x} > 0) \\ \frac{1}{2} \\ 3 \end{cases}$$

$$\Longrightarrow x = \begin{cases} \ln\frac{1}{2} = -\ln 2 \\ \ln 3 \end{cases}$$

$$\therefore \quad x = -\ln 2 \quad \text{OR} \quad x = 3$$